Thinking alone and thinking together

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Non-distributive ascription of belief Consider the following context-sentence pairs:

> *Sam convinced each of her six clients, who do not know each other, that she would build a house for him/her. In reality, Sam was a con artist and built no houses at all.*

(1) (Taken together,) Sam's six clients thought she built six houses for them.

> *Paul just got married, and his cousins Arnie and Beatrice, who have never met, just caught wind of it. Arnie suspects that Paul's husband is rich, and has no other relevant opinions. Beatrice thinks he's a New Yorker (NYer), and has no other relevant opinions.*

(2) (Taken together,) Paul's cousins think he married a rich NYer.

Each of these sentences is true in the context provided. But with small revisions to the context, these sentences become false. For instance, (1) is false if Sam's clients knew that she only had a single lot that can fit a single house, and she promised each client that she would build his/her house on that lot. Likewise, if Arnie thinks Paul married a rich Marylander (MDer), while Beatrice thinks he married a poor NYer, (2) is false, but (3) is true:

(3) (Taken together,) Paul's cousins think he married either a rich MDer or a poor NYer.

Kratzerian belief-summing Say we adopt an event-relative Hintikkan semantics for *believe* as in (4), where $\text{Dox}(e)$ is the set of worlds compatible with the beliefs of the experiencer of e. The denotation for *Arnie thinks Paul married a rich man* will then be as in (5), where $\text{Exp}(e)$ is the experiencer of e. (I assume separation of the external argument as in Kratzer 1996.)

(4) $[\![\text{believe}]\!] = \lambda p \lambda e.\ \forall w \in \text{Dox}(e)[p(w)]$

(5) $\exists e[\text{Exp}(e) = \text{arnie} \wedge \forall w \in \text{Dox}(e)[\text{rich\_man}(w)]]$

Given belief states e_1 and e_2, we must determine what $\text{Dox}(e_1 \sqcup e_2)$ is (where $\sqcup$ is mereological sum). The generalization seems to be roughly as follows: if $\text{Dox}(e_1)$ and $\text{Dox}(e_2)$ are mutually compatible, then $\text{Dox}(e_1 \sqcup e_2) = \text{Dox}(e_1) \cap \text{Dox}(e_2)$. Thus, for (1), each client's belief worlds are agnostic about the other clients getting a house, so their beliefs are mutually compatible and thereby intersected. The result is a set of belief worlds in which each client gets a house. But if $\text{Dox}(e_1)$ and $\text{Dox}(e_2)$ are disjoint, $\text{Dox}(e_1 \sqcup e_2) = \text{Dox}(e_1) \cup \text{Dox}(e_2)$. Thus, if Arnie's belief is that Paul married a rich MDer, and Beatrice's is that he married a poor NYer, their summed belief will be a mixture of rich MDer worlds and poor NYer worlds, making (2) false and (3) true.

As it turns out, this pattern is precisely what we predict if we adopt a (suitably revised) Lewis-Kratzer premise semantics (Lewis 1981, Kratzer 1981), in which worlds are ordered by a set Q of propositions as in (6), where $w_1 \precsim_Q w_2$ iff w_1 is "at least as good" as w_2 with respect to Q:

(6) $w_1 \precsim_Q w_2$ iff $\{p \in Q \mid p(w_1)\} \supseteq \{p \in Q \mid p(w_2)\}$

Let's say that $Q = \{p, q\}$. If p and q are mutually compatible (i.e., $p \cap q \neq \emptyset$), then $\precsim_Q$ will look like Figure 1, where "better" worlds are toward the top, and the set of ideal worlds is circled. In this case the set of ideal worlds is $p \cap q$. But if p and q are *in*compatible, then the ordering is as in Figure 2, where the set of ideal worlds is $p \cup q$. We can therefore adopt the principle (9), where E is a set of belief states, and $\bigsqcup E$ is the sum of the members of E:

(7) $\mathrm{Exp}(\bigsqcup E) = \bigsqcup\{\mathrm{Exp}(e) \mid e \in E\}$, and $\mathrm{Dox}(\bigsqcup E) = \mathrm{Best}_1(\precsim_{\{\mathrm{Dox}(e) \mid e \in E\}})$
(where $\mathrm{Best}_1(\precsim) = \{w \mid \neg\exists w'[w' \prec w]\}$)

If (I) e_a and e_b are Arnie and Beatrice's attitude states, (II) $\mathrm{Dox}(e_a)$ is just the set of worlds in which Paul married a rich man (for simplicity's sake), and (III) $\mathrm{Dox}(e_b)$ is the set of worlds in which he married a NYer, then $\mathrm{Dox}(e_a) \cap \mathrm{Dox}(e_b) \neq \emptyset$, so $\mathrm{Best}_1(\precsim_{\{\mathrm{Dox}(e_a),\mathrm{Dox}(e_b)\}}) = \mathrm{Dox}(e_a) \cap \mathrm{Dox}(e_b)$. Since $\mathrm{Exp}(e_a \sqcup e_b) = a \sqcup b$ and $\mathrm{Dox}(e_a \sqcup e_b) = \mathrm{Dox}(e_a) \cap \mathrm{Dox}(e_b)$, there is a state in which Arnie and Beatrice believe that Paul married a rich NYer. But if $\mathrm{Dox}(e_a)$ and $\mathrm{Dox}(e_b)$ are disjoint, as in the context for (3), then $\mathrm{Best}_1(\precsim_{\{\mathrm{Dox}(e_a),\mathrm{Dox}(e_b)\}}) = \mathrm{Dox}(e_a) \cup \mathrm{Dox}(e_b)$. In this case we (correctly) predict (2) to be false and (3) to be true.

Aboutness Say that Arnie and Beatrice disagree about Mozart's birth date: Arnie thinks it's 1755, and Beatrice knows it's 1756. This irrelevant disagreement should not suffice to render (2) false, but we currently predict it does. After all, all of Arnie's belief-worlds are worlds in which Mozart was born in 1755, while all of Beatrice's are ones where he was born in 1756. Thus, $\mathrm{Dox}(e_a)$ and $\mathrm{Dox}(e_b)$ are disjoint, and we predict a conjunctive claim to be false.

We can fix this by claiming that each belief state is in some sense "about" some situation s (cf. Kratzer 2002), with the set of worlds compatible with this belief state being those worlds compatible with what one believes *specifically about* s. Thus, not every situation about which Arnie has beliefs is such that those beliefs entail that Mozart was born in 1755. The final denotation of (2) will therefore be along the lines of (8), where s^c is an "about" situation determined by context:

(8) $[\![(2)]\!]^c = 1$ iff $\exists e[\mathrm{Exp}(e) = a \sqcup b \wedge \mathrm{about}(e) = s^c \wedge \forall w \in \mathrm{Dox}(e)[\mathrm{rich\_NYer}(w)]]$

This "aboutness" allows us to filter out irrelevant disagreements, while retaining relevant ones: Mozart's birth date is undecided in Arnie and Beatrice's belief states about the situation containing Paul's wedding, but each is opinionated about Paul's husband's wealth and/or hometown.

Using context-sensitive about-situations also helps account for cases where judgments are fuzzy or sensitive to context. For example, (1) and (9) can be mutually compatible:

(9) Each of Sam's clients thought she built a house for him and only him.

Imagine that Sam's six clients each signed an exclusive contract for a house. Thus, (9) is true. But if those clients then file a joint lawsuit, and their lawyer is proposing a sum of money as recompense for Sam's fraudulent activity, the lawyer can truthfully say (1) in tabulating the damages.

Say that the about-situation is always parameterized to the experiencer, perhaps by variable binding. In this case, the denotations of (9) and (1) will be roughly as in (10) and (11), respectively:

(10) $\forall x[\mathrm{client}(x) \rightarrow \exists e[\mathrm{Exp}(e) = x \wedge \mathrm{about}(e) = s^c_x \wedge \forall w \in \mathrm{Dox}(e)[\mathrm{only\_house}(x, w)]]]$

(11) $\exists e[\mathrm{Exp}(e) = \mathrm{the\_clients} \wedge \mathrm{about}(e) = s^c_{\mathrm{the\_clients}} \wedge \forall w \in \mathrm{Dox}(e)[\mathrm{six\_houses}(w)]]$

For each of the clients k_1, k_2, etc., there is a situation s_1, s_2, etc. such that k_n thinks of s_n that k_n is the only one getting a house. Assuming that $s^c_{k_n}$ returns s_n for each client k_n, (10) is true. But if s_n contains the perceived evidence that k_n and no one else is getting a house, there is presumably some situation s'_n that is part of s_n and only contains the perceived evidence that k_n is getting a house, minus the evidence of exclusivity. The clients' beliefs about these subsituations are thus no longer mutually incompatible, just like in the original context for (1). Assuming the about-situation of a summed belief state is the sum of the about-situations of its sub-states, this leads to a reading in which (11) is true: $s^c_{\text{the-clients}}$ is the sum of s'_1, s'_2, etc.

Figures

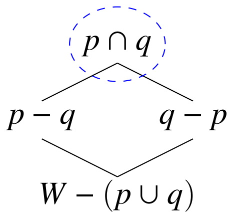

Figure 1

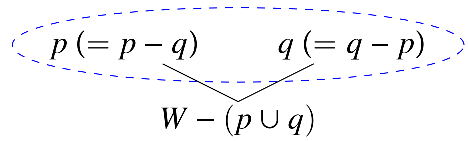

Figure 2

References

Kratzer, Angelika. 1981. The notional category of modality. In H.J. Eikmeyer & H. Rieser (eds.), *Words, worlds, and contexts: New approaches in word semantics*, 38–74. Berlin: de Gruyter.

Kratzer, Angelika. 1996. Severing the external argument from its verb. In J. Rooryck & L. Zaring (eds.), *Phrase structure and the lexicon*, 109–137. Springer Netherlands.

Kratzer, Angelika. 2002. Facts: Particulars or information units? *Linguistics and Philosophy* 25(5/6). 655–670.

Lewis, David. 1981. Ordering semantics and premise semantics for counterfactuals. *Journal of Philosophical Logic* 10(2). 217–234.