Structure preservation in comparatives **Alexis Wellwood (USC)**

Comparisons with nominal *more* involve variable measures both across and within predicates, (1).

(1) a. Mary bought more coffee than John did. **volume**, **weight**, ***temp.**
b. Mary bought more coffees than John did. **number**, ***volume**, ***weight**

(1) help illustrate two generalizations: (i) Monotonicity—measures in nominal comparatives vary, but are structure-dependent (S02, S06); (ii) Number—plurals are only compared in terms of number (H01, BB09). On standard accounts following B73, *more* realizes MUCH-ER in (1a) and MANY-ER in (1b), with correspondingly different measures. I present three reasons to prefer an alternative, univocal account in which *more* realizes MUCH-ER. On this account, (i) and (ii) are captured by a new, stronger condition on the selection of measure functions for MUCH than previously proposed.

Ambiguity+S-monotonicity. Ambiguity accounts will interpret (1) like in (2), where *mc* stands for Mary's coffee and *mcs* Mary's coffees, etc. MUCH with index μ leads to $g(\mu)$ in (2a), a variable assignment of type $\langle e, d\rangle$ (cf. S02, S06, N07, W12, S15, etc), and MANY introduces **number** in (2b) (cf. C76, S84, H85, H01, BP04, S15, etc). The latter assumption guarantees generalization (i).

(2) a. $g(\mu)(mc) > g(\mu)(jc)$ [e.g., $g(\mu) = \mathbf{volume}$, $g'(\mu) = \mathbf{weight}, ...$]
b. $\mathbf{number}(mcs) > \mathbf{number}(jcs)$

Adding a constraint on the selection of μ as in (3) ensures generalization (ii) (S02, S06): assuming that *coffee* is true of portions of coffee and their sums (C75, L83, etc), (3) ensures smaller parts have smaller μ-measures along dimension δ, which rules out assignments like $\mu = \mathbf{temperature}$.

(3) **S-monotonicity**: $\forall x, y \in D$, if $x <_P y$ then $\mu(x) <_\delta \mu(y)$.

Reasons to seek an alternative. First, the cross-linguistic picture suggests that the *much/many* distinction is a surface quirk of English. Generally, the presence or absence of (broadly) plural marking indicates number for an otherwise univocal form: consider Spanish *mucha cerveza/mucha**s** cerveza**s***, French *beaucoup de soupe/beaucoup de biscuit**s***, Italian *molta minestra/molt**i** biscott**i***, Macedonian *mnogu supa/kolac**i***, Mandarin *henduo tang/henduo **kuai** quqi*, and Bangla *onek sup/onek-**gulo** biskuT*. Second, sometimes *much* is restricted to number, yet (3) isn't sufficient to guarantee this. Consider *more furniture* (BB09; cp. *as much furniture*, **as many furniture(s)*), and *Last week, Mary ran to the store as much as John did* (W12). It is easy to concoct scenarios where Mary's furniture weighed more or her running events took more time, but in each case her number was lower. In such cases, **weight** and **duration** would pass (3), and we would incorrectly predict the availability of the opposite of the observed judgments (see BS05 for experimental evidence). Third, **number** meets (3) for plural domains (S02, S06, N07), but this fact is left accidental.

Univocality+A-invariance. These reasons support the desirability of an account that eliminates *many* as a semantic primitive, and in which MUCH can guarantee Number when appropriate. For the first, I assume any sufficient morphophonological rule like (4) that produces the form *many* from MUCH with the nominal plural. The rule must ensure that the relevant grammatical conditions fail to obtain with, e.g., *as much furniture* and *run to the store as much* (cf. *as many time**s***).

(4) MUCH $\rightarrow_{morph}$ *many* / __ PL

This move leaves just the 'weight problem'—e.g., if MUCH were subject only to (3), we would incorrectly predict **weight** to be a permissible value of μ for plurals. To capture Number, we need a constraint that accords with an intuition like the following: **number** values μ for plural domains because only it uniquely characterizes those domains, e.g., only it guarantees that two-membered pluralities (e.g. $a \sqcup b$) 'out-measure' any one-membered plurality (e.g. c). Thus, I propose (5): any admissible μ must assign the same degree to every x and to x's value under any automorphism h.

(5) **A-invariance**: $\forall h \in Aut(\langle D, \sqcup_P \rangle), \forall x \in D, \mu(x) = \mu(h(x))$.

An automorphism is a structure-preserving, bijective function that maps from the domain of a structure onto that same domain (i.e., automorphism = endomorphism + isomorphism), (6). Any h that assigns an element of D to an element not in D fails 'endo'; any h that maps more than one element of D to the same element in D fails bijectivity; and, most importantly, any h that maps the join of two atoms, e.g. $a \sqcup b$, to an atomic individual, a or b, will be non-isomorphic. Thus, the only hs for plural D are ones that map atoms to atoms, doubletons to doubletons, and so on.

(6) $\forall h \in Aut(\langle D, \sqcup \rangle)$: h is a bijection $D \mapsto D$ s.t. $\forall x, y \in D$, $h(x \sqcup y) = h(x) \sqcup h(y)$.

No more weight problem. By (5), **weight** is blocked for *more coffees*, and **duration** is blocked for *run to the park more*. This is so because those dimensions are sensitive to properties of pluralities of coffee individuals and running events apart from their number. For example, let $S = \{a, b, a \sqcup b\}$ and $g = [a \mapsto b, b \mapsto a, a \sqcup b \mapsto a \sqcup b]$ (i.e., an automorphism on S). Suppose further that **weight**(a) = 120lbs and **weight**(b) = 240lbs, and that **number**$(a) = 1$ and **number**$(b) = 1$. Then, **weight**$(a) \neq$ **weight**$(g(a))$ (violating (5)), because $120 \neq 240$, but **number**(a) = **number**$(g(a))$, because $1 = 1$, etc. The same reasoning goes, mutatis mutandis, for the measure of running events.

Two consequences. (i) *many* needn't be treated as a semantic primitive; this raises new questions of analysis wherever it is thought that that expression introduces a cardinality function. (ii) Will A-invariance replace S-monotonicity, or merely supplement it? The answer to this depends on whether A-invariance causes problems for cases like (1a). It is generally, though not always, supposed that the extension of *coffee* has the structure of a join semi-lattice, differing from that of *coffees* in its density and lack of accessible minimal parts (e.g., G12). Suppose we take this suggestion to mean that the extension of coffee is isomorphic to the set of connected subsets (i.e., subintervals) of the interval $(0, 1]$ on the real numbers, ordered by the the subset relation.[1] By (5), exactly the sets that are so ordered in the domain must match those in the range ('endo'), and all of the join relations must be preserved one to one ('iso'); thus, it appears that A-invariance can replace S-monotonicity.

Discussion. The proposed account, on which expressions like *more* uniformly involve a strongly structure-preserving, context-sensitive map to degrees, helps to make sense of certain data in language acquisition and psycholinguistics. First, children appear to acquire *more* along with its constraints across mass and plural nouns simultaneously (O13). This acquisition pattern is predicted on a univocal account, but not on an ambiguity account. Second, children's errors in learning the *much*/*many* contrast suggest that morphology rather than semantics is at play. That is, children extend *much* to plural nouns as late as 7 years 6 months, yet never extend *many* to bare nouns (G85). Finally, the fact that children as well as adults (at least under speeded conditions) use their approximate number system (ANS) to evaluate comparatives with plural nouns (H08, O13) raises questions and potential challenges for accounts that hard-code a semantics in terms of exact cardinality. On the present account, though, we can distinguish both **approximate number** and **exact number** as possible values for μ with plurals. Importantly, while exact numbers are modeled as points and ANS magnitudes as Gaussian distributions, these two 'scale' orderings are isomorphic (e.g., GG92); thus, measures by either will satisfy A-invariance for plural domains.

[1] 'Connected' excludes the Cantor set, which is not measurable (M. Glanzberg, p.c.). There are other options.

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